'90 PT2

Part II: Answer two (2) questions from this part. Show all work. Each question is worth fifteen (15) points.

$\frac{1, 2}{1, 2, 5, 10}$

2 −4 −26 −20

1/2 | 2 −5 −24 −7 10
1 −2 −13 −10
2 −4 −26 −20 0

1. Given: $2x^4 - 5x^3 - 24x^2 - 7x + 10 = 0$

+ − − − 4
+ + − + +

A. 1. Find the maximum number of real positive roots.(2) 2

2. Find the maximum number of real negative roots.(2) 2

B. Find all possible rational roots. (3) ±(1/2, 5, 10, 1, 2, 5/2)

C. Of the possible rational roots find

1. The upper bound (1)
2. The lower bound (1)

D. Solve the equation completely over C,the complex numbers.(5)

2. Using the principle of mathematical induction to show that the following formula is valid for all natural numbers.(15)

$\frac{1}{3} = \frac{1}{2+1} = \frac{1}{3}$ $1/3 + 1/15 + 1/35 + \ldots + 1/(4n^2-1) = n/(2n+1)$ $\therefore$ $1/(4n^2-1) \doteq \frac{n}{2n+1}$

$\frac{1}{4(n+1)^2-1} = \frac{n+1}{2(n+1)^2+1)}$

3. Find all the cube roots of −27i and place your answers in rectangular form. (15)

4. Given the equation: $4x^2 + y^2 + 8x - 2y + 1 = 0$

A. Write the name of the graph.(1)

B. Find the coordinates of the center.(3)

C. Find the coordinates of the foci.(3)

D. Find the coordinates of the vertices.(3)

E. Graph the curve.(5)

$\frac{1}{4n^2+8n+4} \quad \frac{n+1}{2n^2+4n+2+1}$

$\frac{1}{4(n+1)^2-1} = \frac{1}{2n+2+1}$

2n+3

$\frac{1}{4n^2+8n+4} \quad \frac{n+1}{2n^2+4n+4}$

Name Alex Lozupone
Date 12-3-92

Quiz

100

1. Find the sum of the first 1000 terms in the sequence 1,2,3,...

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_n = 500(2+999\cdot 1)$

$S_n = 500(1001)$

$\boxed{S_{1000} = 500500}$ ✓

2. Find the sum of the first five terms of

-1/2 + 3/10 - 9/50 + ...

~~$S_n = \frac{n}{2}(2a+(n-1)d)$~~

~~$S_n = \frac{5}{2}(-1+2(4))$~~

$S_n = \frac{a(1-r^n)}{1-r}$

$S_5 = \frac{-\frac{1}{2}\left(1+\left(\frac{3}{5}\right)^5\right)}{1+\frac{3}{5}}$

$S_5 = \frac{-\frac{1}{2}\left(1+\frac{243}{3125}\right)}{\frac{8}{5}}$

$\frac{-\frac{1}{2}\left(\frac{3368}{3125}\right)}{\frac{8}{5}} = \frac{-\frac{1684}{3125}}{\frac{8}{5}}$

$\boxed{\frac{421}{1250}} = S_5$

$S_5 = \boxed{\frac{-421}{1250}}$ ✓

3. Find the indicated term for the geometric sequence given

2/3, 4/9, 8/27, ... a_8 = $\frac{256}{6561}$

$a_n = a_1 \cdot r^{n-1}$

$a_8 = \frac{2}{3} \cdot \frac{2}{3}^7$

$a_8 = \left(\frac{2}{3}\right)^8 = \boxed{\frac{256}{6561}}$ ✓

4. Find the indicated term in the arithmetic sequence.

6, 11, 16, 21, ... a_{100} = 501

$a_n = a+(n-1)d$

$a_n = 6+(99)(5)$

$a_{100} = 6+495 = \boxed{501}$ ✓

5. Determine the geometric sum.

9/10 + 9/100 + 9/1000 + ...

$S_\infty = \frac{a}{1-r}$

$= \frac{9/10}{1-1/10}$

$= \frac{9/10}{9/10} = \boxed{1}$ ✓

6. Determine the first term of an arithmetic sequence in which the common difference is 5 and the sum of the first 38 terms is 3534.

$d = 5$

$S_{38} = 3534$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$3534 = 19(2a+37\cdot 5)$

$186 = 2a+185$

$2a = 1$

$\boxed{a_1 = \frac{1}{2}}$ ✓

$S_{38} = 19(1+37\cdot 5)$

$S_{38} = (1+185)19$

$S_{38} =$ ✓

Name Alex Lozupone 26 Date 10-1-92

#1,A J. J. Schimmel

1. Factor Completely

a) $3x - 6xy$ $3x(1-2y)$ b) $x^2 - 3x - 10$ $(x-5)(x+2)$

c) $x^2 - y^2$ $(x+y)(x-y)$ d) $16x^2 - 25$ $(4x+5)(4x-5)$

[illegible] $x+1$ $x-1$

e) $x^3 + 8$ $(x+2)(x^2-2x+4)$ f) $x^{10} - 1$ ~~$(x+1)(x-1)(x^4+1)(x^2+1)(x+1)$~~

$(x^5+1)(x^5-1)$

SEE BACK $(x^4+1)(x^2+1)(x+1)^2(x-1)^2$

g) $3x^4 + 3x$ $3x(x+1)(x^2-x+1)$ h) $9m^2 + 42m + 49$ $(3m+7)^2$

$3x(x^3+1)$ $(m+\ )(m+7)$

2. find the quotient and remainder when f(x) is divided by g(x). (Show all work).

$f(x) = x^4 + 4x^2 + x + 2$ $g(x) = x^2 + 2x + 1$

$$
\begin{array}{r}
x^2 - 2x + 7 \\
x^2+2x+1 \,\big)\, x^4 + 0x^3 + 4x^2 + x + 2 \\
-(x^4 + 2x^3 + x^2) \\
\hline
-2x^3 + 3x^2 + x \\
-(-2x^3 - 4x^2 - 2x) \\
\hline
7x^2 + 3x + 2 \\
-(7x^2 + 14x + 7) \\
\hline
-11x - 5
\end{array}
$$

$\boxed{x^2 - 2x + 7 \quad r\ -11x - 5}$

$$
\begin{array}{r}
x^2 - 2x + 7 \\
x^2 + 2x + 1 \\
\hline
x^2 - 2x + 7 \\
2x^3 - 4x^2 + 14x \quad 0 \\
x^4 \quad -2x^3 \quad 7x^2 \\
\hline
x^4 + 4x^2 + 12x + 7 \\
-11x - 5 \\
\hline
x + 2
\end{array}
$$

$7x^4$ $-x$ $-x^2$ 0_4

Given f(x) and One of its zeros find all others zeros.

36
4 32
[illegible]
116
8

$f(x)=x^3-12x^2+44x-48 \quad c=4.$

$$\begin{array}{r|rrrr} 4 & 1 & -12 & +44 & -48 \\ & & 4 & -32 & 48 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

$x^2-8x+12$

$(x-6)(x-2)$

$\frac{-6\pm\sqrt{6^2-4ac}}{2a}$ $\quad \frac{8\pm\sqrt{64-48}}{2}=\frac{8\pm\sqrt{16}}{2}=6,2$

$x=6,2$

$\boxed{6,2}$

$$\begin{array}{r|rrrr} 6 & 1 & -12 & +44 & -48 \\ & & 6 & -36 & 48 \\ \hline & 1 & -6 & 8 & 0 \end{array} \checkmark$$

$$\begin{array}{r|rrrr} 2 & 1 & -12 & +44 & -48 \\ & & 2 & -20 & 48 \\ \hline & 1 & -10 & 24 & 0 \end{array} \checkmark$$

4. Use synthetic division to find f(c).

$f(x)=\overset{64}{x^6}+\overset{64}{4x^4}-6x+5$ find $f(2)$

128

$$\begin{array}{r|rrrrrrr} 2 & 1 & +0 & +4 & +0 & +0 & -6 & +5 \\ & & 2 & 4 & 16 & 32 & 64 & 116 \\ \hline & 1 & 2 & 8 & 16 & 32 & 58 & 121 \end{array}$$

$\boxed{121}$

5. Use synthetic division to find the quotient and remainder of f(x) is divided by g(x).

$f(x)=x^5-x^3+2x^2 \quad g(x)=x+1$

$$\begin{array}{r|rrrrrr} -1 & 1 & +0 & -1 & +2 & 0 & 0 \\ & & -1 & 1 & 0 & -2 & 2 \\ \hline & 1 & -1 & 0 & 2 & -2 & 2 \end{array}$$

$\boxed{x^4-x^3+2x-2 \quad r2}$

$$\begin{array}{r} x^4-x^3+2x-2 \\ x+1 \\ \hline x^4-x^3+2x-2 \\ x^5-x^4-2x \end{array} \checkmark$$

x

6. List all possible rational roots of

$f(x)=x^3-5x^2-x+5$

$\frac{\{5,1\}}{\{1\}}$ $\quad -5,-1,1,5$

b) find all the roots.

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & +5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -1 & +5 \\ & & 5 & 0 & -5 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

x^2-1

$(x+1)(x-1)$

$\boxed{-1,1,5}$

ALEX COZFOM

Stuyvesant High School
Abraham Baumel, Principal

Mathematics Department
Dr. R. Rothenberg, APS

Final Examination - ME81
January 1990

Part I: Do all problems. Questions 1-10 four (4) points each. Questions 11-15 six (6) points each.

1. If $g(x) = \frac{x-4}{2x+1}$ find g^{-1}, the inverse of g(x).

3 | 1 2
5
1 5 . . .

-3 | 1 5 6
-3 -6
1 2 0

2. What is the multiplicity of x+3 as a factor of

$$x^4 + 11x^3 + 33x^2 + 9x - 54 \ ?$$

3

-3 | 1 11 33 9 -54
-3 -24 -27 54
1 8 9 +18 0

-3 | 1 8 9 18
-3 -15 -18
1 5 6 0

3. What is the quotient and remainder when $x^6 + 2x^4 + 3x^2 - x + 2$ is divided by x-2?

2 | 1 0 2 0 3 -1 2
2 4 12 24 54 106
1 2 6 12 27 53 108

$x^5 + 2x^4 + 6x^3 + 12x^2 + 27x + 53$ r 108

4. Write a polynomial equation of lowest possible degree with real coefficients two of whose roots are -2 and 4 -3i .

5. Find the product of $4(\cos 105^\circ + i \sin 105^\circ) \times 3(\cos 75^\circ + i \sin 75^\circ)$. Place your answer in rectangular form.

6. Find the eighth term in the expansion of $(3x - y^3)^9$.

7. Find the sum of this infinite series 2/3, -1/3, 1/6, -1/12,...

8. What is the sum of the first 20 positive, non-zero multiples of 4?

9. The equation of the asymptotes of the hyperbola $x^2 - 4y^2 = 16$ is

   a) $y = \pm 4x$ b) $y = \pm (1/4) x$ c) $y = \pm 2x$ d) $y = \pm (1/2) x$

10. Write the equation of a parabola whose vertex is the origin and whose equation of the directrix is x-3=0.

Questions 11-15 are worth 6 points each. Answer all parts.

11. Find the domain and range for $f = \{(x,y) \mid y = \sqrt{25 - x^2}\}$.

12. Find the sum and the product of all the roots $x^5 = 2x^3 + 8x + 16$

13. Use DeMoivre's Theorem to find $(-\sqrt{3} - i)^7$. Place answer in rectangular form.

14. Find the coordinates of the center and the radius of the circle described by this equation $x^2 + y^2 - 6x + 10y + 30 = 0$

15. Find the length of the major axis, the coordinates of the foci and the eccentricity of the ellipse descibed by this equation.

$$x^2 + 25y^2 = 25$$

1 How many terms of the A. P. $-5, -1, 3, \ldots$ must be added to give a sum of 400?

2 Find the sum of all positive integers less that 200 that are multiples of 7.

3 Find the first two terms of an A. P. in which the third term is 14 and the ninth term is -1.

4 For what value(s) of t will $t - 2$, $2t - 6$, and $4t - 8$, in this order, form an A. P.?

5 For what real value(s) of k (if any) will -1 be the arithmetic mean of k and k^2?

2842

$S_{27} = 14(14 + 27 \cdot 7) = 2013$ = ~~406~~

~~t-2= =2t~~ $2t - 6 - t + 2 = 4t - 8 - 2t + 6$

$t - 4 = 2t - 2$

$t = -2$

6 Find three numbers in A. P. whose sum is 27 and whose product is 288.

7 The first four terms of an A. P. are r, s, t, u. Show that $r + u - s = t$.

8 The average of the n terms $a_1, a_2, a_3, \ldots, a_n$ is $\frac{1}{n}(a_1 + a_2 + a_3 + \cdots + a_n)$.

9 Show that if these terms are in arithmetic progression the average of the first and last terms is the same as the average of all n terms.

10 Show that $S_n = \sum_{k=1}^{n} [a + (k - 1)d]$ and then use the theorems stated in Exercises 47, 48, and 49, page 79, and in Exercise 14, page 73, to prove that $S_n = \frac{n}{2}[2a + (n - 1)d]$.

11 A woodsman stacks $8k + 15$ logs in such a way that there are k layers with 8 logs in the top layer. Each layer below contains one more log than the layer immediately above. Find the number of logs.

12 There are k animal feeding stations arranged in a line with a supply hut. The stations are s yards apart and the nearest is t yards from the hut. An attendant carries n bags of feed, one at a time, to each feeding station. How far will he have traveled when he arrives back at the hut after servicing all stations?

13 Insert two real numbers between 2 and 9 so that the first three terms form an A. P. and the last three terms form a G. P.

14 A car purchased for \$2500 depreciates 15% in value every year. Find the value of the car at the end of a four-year period. 2500 - 15

1305

15 Find the second term of an A. P. whose first term is 2 and whose first, third, and seventh terms form a G. P.

16 The sum of the first and last terms of a G. P. of fourteen real terms is 7. The fifth term is the mean proportional between the second and last terms. Find the third term.

17 One-third of the air in a tank is removed with each stroke of a vacuum pump. What part of the original amount of air remains in the tank after five strokes?

18 Show that the reciprocals of the terms of a G. P. ($a \neq 0$, $r \neq 0$) form a G. P.

19 Prove: If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, the arithmetic mean of a^2 and b^2 is not less than the absolute value of their geometric mean.
[*Hint:* $(a - b)^2 \geq 0$.]

For every sequence which is both an A. P. and a G. P., determine the common **(a)** difference; **(b)** ratio.

21 Given a circle of radius t. Show that the area of the inscribed regular hexagon is a mean proportional between the areas of the inscribed and circumscribed equilateral triangles.